the symbol sacred to his village (gens), there was no attempt to coerce the child to observe any rite or to direct its mind to think upon tribal beliefs.

A child gifted with an inquiring mind would be apt to ask questions concerning the symbols and the customs attending them of his father or mother and if his parents belonged to the thinking class, they might put the child's thoughts on the trail that in time would lead him to a knowledge and an understanding of the religious beliefs expressed in the tribal rites, otherwise, the boy or girl would grow up in unquestioning ignorance of the truths hidden within the dramatic rites of the tribe.

The sacred legend of the Omaha lays stress upon 'thought.' 'And the people thought,' was always the prelude to any change for the betterment of the tribe. The Indian thinker became the Indian seer, but, the man must seek for himself the path to that height.

THE CORRELATION OF POTASSIUM AND MAGNESIUM, SODIUM AND IRON, IN IGNEOUS ROCKS

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In its modern development the science of petrology is devoted, in great part, to the physico-chemical study of igneous rocks, regarded as congealed solutions, and of the conditions of separation of their constituent minerals. Distinct from, and in some respects more general than, this, is the study of the distribution of the chemical elements of which they are composed.

This distribution may be spatial; that is, according to the occurrence of groups of rocks, or rock series, over different areas of the earth, each with its dominant chemical characters. Such districts are known as petrographic provinces or comagmatic regions. Their study involves the consideration of the original homogeneity or heterogeneity of the globe, as well as, possibly, certain tectonic factors, as suggested by Harker, Becke, and others, and does not concern us here.

Or the distribution may be according to a chemical correlation, expressed by the presence of certain pairs or groups of elements, which vary together, however they may be spatially distributed. In other words, in rock magmas, solidified as igneous rocks, as well as in their constituent minerals, certain of the elements are found to be generally and persistently associated together, the one being abundantly present

when the amount of its correlate is high, and the converse, but not usually so when the rock magma is dominantly characterized by the abundance of another, non-correlated element, in which case another element, correlate with it, will appear as the co-dominant one.

This phase of petrology, which is of a complexity probably much greater than is superficially apparent both as to character and cause, has, as yet, received comparatively little attention. J. H. L. Vogt,[1] J. F. Kemp,[2] L. de Launay,[3] and W. F. Hillebrand[4] have discussed the subject, but have confined their observations almost wholly to the distribution of the elements according to the relative amount of silica (silicity) shown by the igneous rocks.

A few years ago I showed[5] that these relations are far more complex than the simple one based on the silicity. Thus it is clearly obvious from a study of rocks and rock analyses that, among other relations, lithium, zirconium, the elements of the rare earths, glucinum and others, are most abundant in dominantly sodic magmas, irrespective of their silicity; barium apparently in those dominantly potassic; titanium, vanadium, nickel and manganese in those high in iron; and chromium and platinum in those high in magnesium. It was also stated that "soda not uncommonly tends to vary with the iron oxides, while potash shows similar relations to magnesia."

The recent compilation of a second edition of *A Collection of the Chemical Analyses of Igneous Rocks*,[6] embracing all those published between 1883 and 1914 (numbering nearly 10,000), and soon to be published, has added greatly to the data available. This cumulative evidence now greatly strengthens, if it does not quite establish, the view that in igneous magmas potassium and magnesium on the one hand, and sodium and iron on the other, tend to vary together.

The scope of this paper does not permit the presentation in full or in detail of the data in regard to this or other correlations which have been noted, but a short statement may be of interest, in advance of a more extended publication, as calling attention to a phase of chemical petrology which has been somewhat neglected, but which would seem to be of fundamental importance.

The evidence is of two kinds; mineralogical and petrographical.

Among the important mineral groups, found in igneous rocks, which contain the elements in question, the law enunciated above holds good almost without exception. The sodic pyroxenes and amphiboles, such as aegirite, acmite, riebeckite, arfvedsonite, barkevikite, kaersutite, and basaltic hornblende, all show, along with practical absence of potassium, a strong preponderance of iron over magnesium, the latter usually

being present in these only in traces or at least in very small amounts. The few reliable analyses of augites from highly potassic rocks, on the other hand, show little soda, but a marked preponderance of magnesium over iron. It may be noted, incidentally, that there are no natural potassic pyroxenes or amphiboles corresponding to the sodic ones.

On the other hand in the ferromagnesian micas, biotite and phlogopite, the alkali metal is entirely or almost entirely potassium, the amount of sodium being generally negligible. In these there is usually a very marked predominance of magnesium over iron. Only in the potash mica, lepidomelane, which is characterized by high ferric iron, does iron dominate over magnesium, but it is to be noted that this variety seems to be confined to the sodic rocks. It is also noteworthy that there are no true soda micas, except paragonite, which is only known in metamorphic rocks. It may be added that the potassium-lithium mica, zinnwaldite or cryophyllite, carries much more iron than magnesium, and attention was called above to the fact that lithium is usually associated with sodic rocks.

Of the multitudinous examples of the general law furnished by igneous rocks only a few prominent ones can be given here.

Highly potassic rocks are very rare, but the few known regions are characterized by high magnesium relative to iron. Such are the wyomingite, orendite and madupite of the Leucite Hills, Wyoming; the jumillite and verite of Murcia, Spain; the leucitic rocks of Celebes and of the Gaussberg, in the Antarctic; many of the leucitic rocks of the volcanoes of Western Italy (through here the dominance of magnesium over iron is not so pronounced). There are also many other scattering examples of syenites, shonkinites, minettes, leucitites, leucite tephrites, and other dopotassic rocks. Indeed the same general relation holds good even in such rocks as peridotites, pyroxenites and hornblendites, in which the amounts of magnesium and iron are very high and those of the alkalies very low.

Of the converse case, high sodium with high iron relative to magnesium, very striking examples are the highly sodic rocks of the Julianehaab District in Southern Greenland, in which magnesium is usually present only in traces and often wholly absent; the ijolites and other nephelite rocks of the Kola Peninsula in Finland; the nephelite syenites and related rocks of Ontario, Norway, the Transvaal, Portugal, Brazil, and other regions; and the sodic lavas of Pantelleria, Madagascar, and the Great Rift Valley in East Africa. Indeed illustrations of the law are so abundant among the highly sodic rocks as to make selection difficult.

In the figure is given the result of plotting nearly two hundred superior

analyses of rocks from all over the globe. To avoid complications due
to the presence of considerable lime, a subject for further investigation,
only peralkalic and some domalkalic rocks were selected, chiefly among
the perpotassic, dopotassic, and persodic subrangs. The analyses be-
longing to the sodipotassic and dosodic subrangs are so numerous that
only a very few of the many available were used. These analyses repre-
sent rocks from Greenland, Canada, Montana, Colorado, Wyoming,

MᴳO

FᴇO

Nᴀ₂O   K₂O
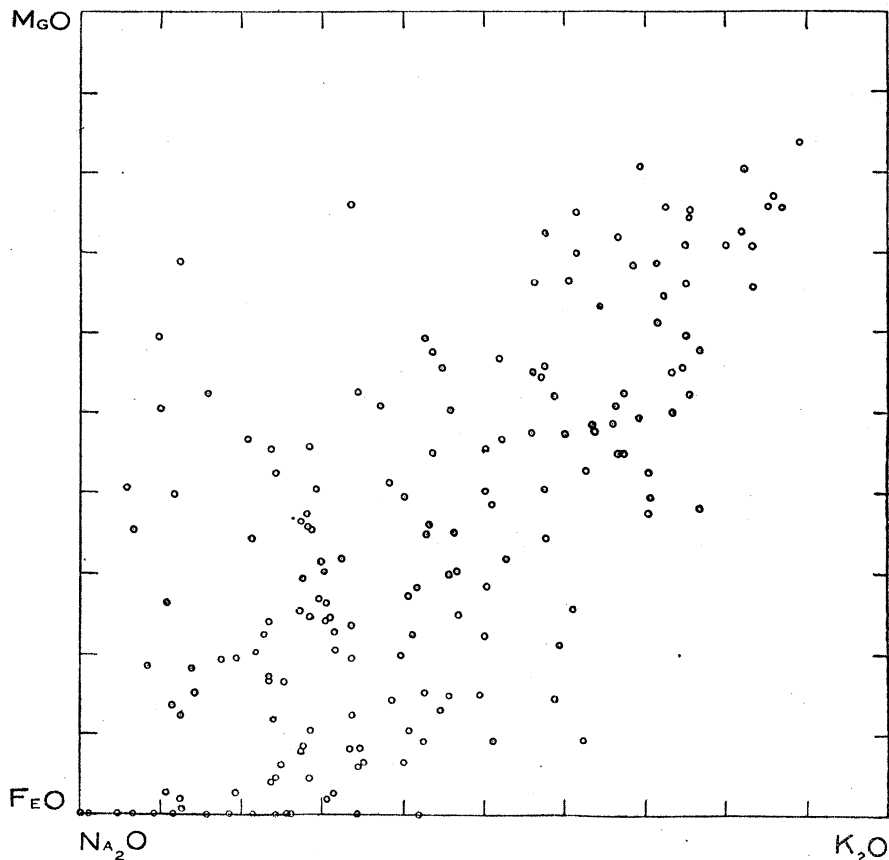
Scotland, Norway, Finland, Bohemia, Italy, Madagascar, Celebes,
Africa, Australia, Antartica and other localities, so that, as a whole,
variations due to regional peculiarities are eliminated.

It may be added that the silica percentages of the rocks represented
by these analyses vary from 39.24 to 73.68, thus covering nearly the
entire range.

The clustering in a narrow band, extending diagonally from high
soda and iron in the lower left quadrant, to high potash and magnesia

in the upper right, with the practical absence of analyses in the upper left and lower right quadrants, is very evident.

The chemical composition of igneous rocks is so complex and the number of their chemical constituents is so great; the conditions of differentiation and solidification are so varied and so complicated; and the knowledge of the application of the laws of physical chemistry to them is as yet so meager by reason of the lack of data, that some irregularities and apparent exceptions are to be expected. Furthermore there are other correlative relations which have not been dealt with here, and it is conceivable or indeed probable that one or more of these may under certain conditions, whether of composition, differentiation or solidification, supersede the one which is the subject of the present discussion without invalidating the truth of this.

It would be premature to enter here into a discussion of the causes of the correlation of these four elements. Any such would necessitate the consideration of the other correlations which have been detected. It may be said, however, that even though such concomitant variations may be due to similar solubility relations, or some such physico-chemical factor, yet that such similarities are themselves presumably due to certain intimate relationships between the elements which are commonly and somewhat loosely called affinities.

[1] J. H. L. Vogt., *Zs. prakt. Geol.*, 1898, 326.
[2] J. F. Kemp., *Ore Deposits*, 34–37 (1900).
[3] L. de Launay, *La Science Géologique*, 637, 1905; *Gîtes Mineraux*, 1, 46–51 (1913).
[4] W. F. Hillebrand., *Bull. U. S. Geol. Surv.*, 305, 21 (1907).
[5] H. S. Washington., *Trans. Amer. Inst. Min. Eng.*, 1908, 749–767.
[6] H. S. Washington., *Prof. Paper U. S. Geol. Surv.*, 14, 1903.

THEOREM CONCERNING THE SINGULAR POINTS OF ORDINARY LINEAR DIFFERENTIAL EQUATIONS

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In earlier papers I have considered the effect of a linear transformation of dependent variables upon the solutions of ordinary linear differential equations in the vicinity of a singular point.

This type of transformation led me to the notion of *equivalence* which is fundamentally important for the classification of singular points.

Ordinary linear differential equations also preserve their form under an arbitrary transformation of the independent variable. I shall prove here that this second type has no additional significance for the purposes